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Compton scattering from a pion and off–shell effects*

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Abstract

We discuss the most general form of the electromagnetic vertex of an off–shell pion. The framework of chiral perturbation theory is used to illustrate the representation dependence of the electromagnetic three–point Green's function. For that purpose we discuss the concept of field transformations which, in comparison with the standard Gasser and Leutwyler Lagrangian, generate equivalent Lagrangians containing additional terms at order p^4 proportional to the lowest–order equation of motion. We consider Compton scattering from a pion to show that calculations involving different off–shell effects in the s– and u–channel pole diagrams may nevertheless lead to the same on–shell Compton scattering amplitude. This is a result of the equivalence theorem which states that two equivalent Lagrangians predict the same S–matrix elements even though they may generate a different off–shell behaviour of Green's functions. We conclude that off–shell effects are not only model dependent but also representation dependent.

I. INTRODUCTION

A covariant description of almost any electromagnetic process involving pions will, in principle, require the off–shell electromagnetic vertex of a pion [1] which, in general, is expected to be more complicated than the free vertex. Let us consider a few examples in order to illustrate this point. The t–channel pole diagram of pion photo– and electroproduction off a nucleon contains a virtual pion which interacts with the photon and then emerges as an on–shell pion. Similarly, pion–nucleon bremsstrahlung will involve four pole diagrams where the photon is radiated off the external pion lines. One pion line is off shell, the other on shell. A situation where both pion lines are off mass shell is encountered in meson exchange current contributions to electron scattering from nuclei. Clearly, the above and many other

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examples suggest that a systematic study of off–shell effects in electromagnetic interactions is important.

The purpose of this talk is to deal with the following three issues:

1. What is the most general form of the electromagnetic vertex of an off–shell pion?
2. How does this vertex show up in the calculation of an invariant amplitude such as, e.g., the Compton scattering amplitude?
3. Is it, in principle, possible to obtain information about off–shell contributions from experimental data?

In order to answer these questions we shall make use of the framework of chiral perturbation theory (ChPT) for mesons [2–4]. Use of a specific theory in the calculation of a simple process, namely $\gamma\pi^+ \to \gamma\pi^+$, will serve us to understand the nature of off–shell contributions. The interpretation of the result, however, is general and will not depend on the fact that we have performed a calculation at $O(p^4)$ in the momentum expansion.

II. ELECTROMAGNETIC VERTEX OF AN OFF–SHELL PION

In this section we shall formally introduce the concept of *form functions* by parameterizing the electromagnetic three–point Green's function of a pion. We deliberately distinguish between *form factors* and *form functions*, since form factors correspond to observables, which is not necessarily true for form functions. Then a discussion of some general properties of these form functions, resulting from the application of symmetry principles, follows.

Let us consider the momentum–space Green's function of two unrenormalized pion field operators $\pi^+(y)$ and $\pi^-(z)$ coupled to the electromagnetic current operator $J^\mu(x)$, [1]

$$\begin{aligned}&(2\pi)^4\delta^4(p_f-p_i-q)G^\mu(p_f,p_i) = \\ &\int d^4x\, d^4y\, d^4z\, e^{-i(q\cdot x-p_f\cdot y+p_i\cdot z)} <0|T\left(J^\mu(x)\pi^+(y)\pi^-(z)\right)|0>,\end{aligned} \tag{2.1}$$

where p_i and p_f are the four–momenta corresponding to the pion lines entering and leaving the vertex, respectively, and $q = p_f - p_i$ is the momentum transfer at the vertex. The renormalized three–point Green's function G^μ_R is defined as [5]

$$G^\mu_R(p_f,p_i) = Z_\phi^{-1} Z_J^{-1} G^\mu(p_f,p_i), \tag{2.2}$$

where Z_ϕ and Z_J are renormalization constants.[2] The irreducible, renormalized three–point Green's function is then given by

$$\Gamma^{\mu,irr}_R(p_f,p_i) = (i\Delta_R(p_f))^{-1} G^\mu_R(p_f,p_i)(i\Delta_R(p_i))^{-1}, \tag{2.3}$$

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[1] $\pi^{+/-}(x)$ destroys a $\pi^{+/-}$ or creates a $\pi^{-/+}$.

[2] In fact, $Z_J = 1$ due to gauge invariance [5].

where $\Delta_R(p)$ is the full, renormalized propagator. From a perturbative point of view, $\Gamma_R^{\mu,irr}$ is made up of those Feynman diagrams which cannot be disconnected by cutting any one single internal line.

We shall now discuss some model–independent properties of $\Gamma_R^{\mu,irr}(p_f,p_i)$ which follow from general symmetry considerations, such as Lorentz covariance, time–reversal and charge–conjugation symmetry, as well as gauge invariance.

1. Imposing Lorentz covariance, the most general parameterization of $\Gamma_R^{\mu,irr}$ can be written in terms of two independent four–momenta, $P^\mu = p_f^\mu + p_i^\mu$ and $q^\mu = p_f^\mu - p_i^\mu$, respectively, multiplied by Lorentz–scalar form functions F and G depending on three scalars, e.g., q^2, p_i^2, p_f^2,
$$\Gamma_R^{\mu,irr}(p_f,p_i) = (p_f+p_i)^\mu F(q^2,p_f^2,p_i^2) + (p_f-p_i)^\mu G(q^2,p_f^2,p_i^2). \tag{2.4}$$

2. Imposing time–reversal symmetry one finds the following properties of the form functions,
$$F(q^2,p_f^2,p_i^2) = F(q^2,p_i^2,p_f^2), \quad G(q^2,p_f^2,p_i^2) = -G(q^2,p_i^2,p_f^2). \tag{2.5}$$
In particular, from Eq. (2.5) we conclude that $G(q^2,M^2,M^2)=0$. This, of course, corresponds to the well–known fact that a spin–0 particle has only one electromagnetic form factor, $F(q^2)$.

3. Using the charge–conjugation properties $J^\mu \to -J^\mu$ and $\pi^+ \leftrightarrow \pi^-$, it is straightforward to see that form functions of particles are just the negative of form functions of antiparticles. In particular, the π^0 does not have any electromagnetic form functions even off shell, since it is its own antiparticle.

4. The equal–time commutation relations of the electromagnetic charge–density operator with the pion field operators,
$$\begin{aligned}
&[J^0(x),\pi^-(y)]\delta(x^0-y^0) = \delta^4(x-y)\pi^-(y),\\
&[J^0(x),\pi^+(y)]\delta(x^0-y^0) = -\delta^4(x-y)\pi^+(y),
\end{aligned} \tag{2.6}$$
and current conservation at the operator level, $\partial_\mu J^\mu(x)$, are the basic ingredients for obtaining Ward–Takahashi identities [6,7] relating Green's functions which differ by one insertion of the electromagnetic current operator. In particular, one obtains for the electromagnetic vertex
$$q_\mu \Gamma_R^{\mu,irr}(p_f,p_i) = \Delta_R^{-1}(p_f) - \Delta_R^{-1}(p_i). \tag{2.7}$$
Inserting the parameterization of the irreducible vertex, Eq. (2.4), into the Ward–Takahashi identity, Eq. (2.7), the form functions F and G are constrained to satisfy
$$(p_f^2-p_i^2)F(q^2,p_f^2,p_i^2) + q^2 G(q^2,p_f^2,p_i^2) = \Delta_R^{-1}(p_f) - \Delta_R^{-1}(p_i). \tag{2.8}$$
From Eq. (2.8) it can be shown that, given a consistent calculation of F, the propagator of the particle, Δ_R, as well as the form function G are completely determined (see Appendix A of Ref. [8] for details). The Ward–Takahashi identity thus provides an important consistency check for microscopic calculations.

III. CHIRAL PERTURBATION THEORY FOR MESONS

In this section we shall give a short introduction to those aspects of chiral perturbation theory [2–4] which are relevant for our discussion of off–shell Green's functions. In particular, the concept of field transformations is discussed in quite some detail since it turns out to be important for the interpretation of form functions.

Chiral perturbation theory provides a systematic framework for describing interactions between the members of the low–energy pseudoscalar octet (π, K, η) which are regarded as the Goldstone bosons of spontaneous symmetry breaking in QCD from $SU(3)_L \times SU(3)_R$ to $SU(3)_V$. The effective Lagrangian of ChPT is organized as a sum of terms with an increasing number of covariant derivatives and quark mass terms,

$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4 + \ldots, \tag{3.1}$$

where the subscripts refer to the order in the momentum expansion. Covariant derivatives and quark mass terms are counted as $O(p)$ and $O(p^2)$, respectively, in the power counting scheme. A systematic classification of Feynman diagrams in the framework of an effective Lagrangian is made possible by Weinberg's power counting scheme [2] which, loosely speaking,[3] can be interpreted as follows. Given a general diagram calculated, e.g., with the Lagrangian of Eq. (3.1), a rescaling of all external momenta $p \rightarrow tp$ and of masses $M^2 \rightarrow t^2 M^2$ leads to the following behaviour of the corresponding invariant amplitude[4]

$$\mathcal{M}(tp, t^2M^2) = t^D \mathcal{M}(p, M^2).$$

The scaling power D is determined by

$$D = 2 + \sum_{n=2,4,\ldots} (n-2) N_n + 2 N_L, \tag{3.2}$$

where N_n refers to the number of vertices for which the number of covariant derivatives n_1 and twice the number of quark mass terms or field strength tensors n_2 adds up to n, and where N_L denotes the number of loops. For small enough external momenta and masses which, of course, can only be controlled theoretically, diagrams with large dimensions D are expected to be less important. According to Eq. (3.2), at $O(p^2)$ one has to consider tree–level diagrams constructed entirely with vertices from $\mathcal{L}_2$, at $O(p^4)$ one has to take account of tree–level diagrams with one vertex from $O(p^4)$ and an arbitrary number of vertices from $O(p^2)$ or one–loop diagrams with vertices from $\mathcal{L}_2$.

The lowest–order Lagrangian is given by [4]

$$\mathcal{L}_2 = \frac{F_0^2}{4} Tr\Big(D_\mu U (D^\mu U)^\dagger + \chi U^\dagger + U \chi^\dagger\Big), \quad U(x) = \exp\left(i\frac{\phi(x)}{F_0}\right), \tag{3.3}$$

where

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[3] We do not address the question of renormalization.

[4] Polarization vectors also have to be rescaled, $\epsilon \rightarrow t\epsilon$.

$$\phi(x) = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta \end{pmatrix}. \tag{3.4}$$

The quark mass matrix which in QCD generates an explicit breaking of chiral symmetry is contained in $\chi = 2B_0\, diag(m_u, m_d, m_s)$. B_0 is related to the quark condensate $<\bar{q}q>$, $F_0 \approx 93$ MeV denotes the pion–decay constant in the chiral limit. The covariant derivative $D_\mu U = \partial_\mu U + ieA_\mu[Q, U]$, where $Q = diag(2/3, -1/3, -1/3)$ is the quark–charge matrix, $e > 0$, generates a coupling to the electromagnetic field A_μ. Finally, the equation of motion (EOM) obtained from $\mathcal{L}_2$ reads

$$\mathcal{O}^{(2)}_{EOM}(U) = D^2UU^\dagger - U(D^2U)^\dagger - \chi U^\dagger + U\chi^\dagger + \frac{1}{3}Tr\left(\chi U^\dagger - U\chi^\dagger\right) = 0. \tag{3.5}$$

The most general structure of $\mathcal{L}_4$ was first written down by Gasser and Leutwyler (see Eq. (6.16) of Ref. [4]),

$$\mathcal{L}_4 = L_1\left(Tr(D_\mu U(D^\mu U)^\dagger)\right)^2 + \ldots, \tag{3.6}$$

and introduces 10 physically relevant low–energy coupling constants L_i.

Before turning to the Compton scattering process we still have to discuss the important concept of field transformations [9–11]. For that purpose we introduce a field redefinition,

$$U' = exp(iS)U = U + iSU + \ldots, \tag{3.7}$$

and demand the same properties of the new fields U' as of U. For example, since both, U and U', are $SU(3)$ matrices we conclude that $S = S^\dagger$ and $Tr(S) = 0$. Furthermore, imposing the constraints of chiral symmetry, charge conjugation and parity it can be shown that two generators exist at $O(p^2)$ (see Ref. [11] for details).

What is the consequence of working with U' instead of U? Inserting U' into $\mathcal{L}$ of Eq. (3.1) results in

$$\mathcal{L}(U) \to \mathcal{L}(U') = \mathcal{L}_2(U) + \Delta\mathcal{L}_2(U) + \mathcal{L}_4(U) + O(p^6), \tag{3.8}$$

where

$$\Delta\mathcal{L}_2(U) = \text{tot.div.} + \underbrace{\frac{F_0^2}{4}Tr(iS\mathcal{O}^{(2)}_{EOM})}_{O(p^4)} + \underbrace{O(S^2)}_{O(p^6)}. \tag{3.9}$$

The total divergence has no dynamical significance and can thus be dropped. The second term of Eq. (3.9) is of $O(p^4)$ and leads to a "modification" of $\mathcal{L}_4$ [8,11],

$$\mathcal{L}_4^{off-shell} = \beta_1 Tr(\mathcal{O}^{(2)}_{EOM}\mathcal{O}^{(2)\dagger}_{EOM}) + \beta_2 Tr((\chi U^\dagger - U\chi^\dagger)\mathcal{O}^{(2)}_{EOM}). \tag{3.10}$$

By a simple redefinition of the field variables one generates an infinite set of "new" Lagrangians depending on two parameters β_1 and β_2. That all these Lagrangians describe the same physics will be illustrated in the next section.

IV. THE COMPTON SCATTERING AMPLITUDE

In this section we shall consider the invariant amplitude for the process $\gamma(\epsilon,k)+\pi^+(p_i)\to\gamma'(\epsilon',k')+\pi^+(p_f)$ at $O(p^4)$ in the framework of the effective Lagrangians of Eqs. (3.3), (3.6) and (3.10). The first discussion of this process in standard ChPT ($\beta_1=\beta_2=0$) can be found in Refs. [12,13]. Our main interest here is to investigate the influence of the additional terms of Eq. (3.10) on a) the form functions of the pion and b) the total Compton scattering amplitude. In particular, we want to find out whether the empirical Compton scattering amplitude can be used to obtain information about the form functions of the pion.

The most general, irreducible, renormalized three–point Green's function (see Eq. (2.4)) at $O(p^4)$, compatible with the constraints imposed by approximate chiral symmetry, was derived in Ref. [8]. For positively charged pions and for real photons ($q^2=0, q=p_f-p_i$) it has the simple form

$$\Gamma^{\mu,irr}_R(p_f,p_i)=(p_f+p_i)^\mu\left(1+16\beta_1\frac{p_f^2+p_i^2-2M_\pi^2}{F_\pi^2}\right), \tag{4.1}$$

and the corresponding renormalized propagator is given by

$$i\Delta_R(p)=\frac{i}{p^2-M_\pi^2+\frac{16\beta_1}{F_\pi^2}(p^2-M_\pi^2)^2+i\epsilon}. \tag{4.2}$$

Note that Eqs. (4.1) and (4.2) satisfy the Ward–Takahashi identity, Eq. (2.7). Clearly, the parameter β_1 is related to the deviation from a "pointlike" vertex, once one of the pion legs is off shell. The electromagnetic vertex and the propagator are both independent of the parameter β_2 of Eq. (3.10). Eqs. (4.1) and (4.2) have to be compared with the result of the usual representation of ChPT at $O(p^4)$. In this case the vertex at $q^2=0$ is independent of p_f^2 and p_i^2, $\Gamma^{\mu,irr}_R(p_f,p_i)=(p_f+p_i)^\mu$. Furthermore, the renormalized propagator is simply given by the free propagator.

We shall now address the question how the parameter β_1 enters the Compton scattering amplitude [14]. For that purpose we subtract the ordinary calculation of the pole terms using free vertices from the corresponding calculation with off–shell vertices and interpret the result as being due to off–shell effects. Similar methods have been the basis of investigating the influence of off–shell form functions in various reactions involving the nucleon, such as proton–proton bremsstrahlung [15], electron–nucleus scattering [16], or virtual Compton scattering [17]. For Compton scattering from a pion the result is found to be,[5]

$$\begin{aligned}\Delta M_P &= \underbrace{M_P(\beta_1\neq 0)}_{\text{incl. off–shell effects}} - \underbrace{M_P(\beta_1=0)}_{\text{ordinary calc.}}\\ &= -ie^2\frac{64\beta_1}{F_\pi^2}\left(\underbrace{p_f\cdot\epsilon'\,p_i\cdot\epsilon}_{\text{s channel}}+\underbrace{p_f\cdot\epsilon\,p_i\cdot\epsilon'}_{\text{u channel}}\right).\end{aligned} \tag{4.3}$$

[5] Of course, using Coulomb gauge $\epsilon^\mu=(0,\vec\epsilon)$, $\epsilon'^\mu=(0,\vec{\epsilon'})$, and performing the calculation in the lab frame ($p_i^\mu=(M_\pi,0)$), the additional contribution vanishes, since $p_i\cdot\epsilon=p_i\cdot\epsilon'=0$. However, this is a gauge–dependent statement and thus not true for a general gauge.

We shall now demonstrate that Eq. (4.3) cannot be used for a unique extraction of the form functions from experimental data. In order to see this we have to realize that Eq. (4.3) is not yet the complete modification of the total amplitude. The reason is that the very same term in the Lagrangian which contributes to the off–shell electromagnetic vertex also generates a two–photon contact interaction. This can be seen by inserting the appropriate covariant derivative into Eq. (3.10) and by selecting those terms which contain two powers of the pion field as well as two powers of the electromagnetic field. From the first term of Eq. (3.10) one obtains the following $\gamma\gamma\pi\pi$ interaction term

$$\begin{aligned}\Delta\mathcal{L}_{\gamma\gamma\pi\pi} = \frac{16\beta_1 e^2}{F_\pi^2}&\Big(-A^2[\pi^-(\Box + M_\pi^2)\pi^+ + \pi^+(\Box + M_\pi^2)\pi^-] \\ &+(\partial\cdot A + 2A\cdot\partial)\pi^+(\partial\cdot A + 2A\cdot\partial)\pi^-\Big)\,. \end{aligned} \tag{4.4}$$

For real photons Eq. (4.4) translates into a contact contribution of the form

$$\Delta\mathcal{M}_{\gamma\gamma\pi\pi} = ie^2\frac{64\beta_1}{F_\pi^2}(p_f\cdot\epsilon'\,p_i\cdot\epsilon + p_f\cdot\epsilon\,p_i\cdot\epsilon'), \tag{4.5}$$

which precisely cancels the contribution of Eq. (4.3). At first sight the second term of Eq. (3.10) also seems to generate a contribution to the Compton scattering amplitude. However, after wave function renormalization this term drops out (see Ref. [14] for details). We emphasize that all the cancellations happen only when one consistently calculates off–shell form functions, propagators and contact terms, and properly takes renormalization into account. Thus the Lagrangian of Eq. (3.8) which represents an equivalent form to the standard Lagrangian of ChPT yields the same Compton scattering amplitude while, at the same time, it generates different off–shell form functions. Clearly, this illustrates why there is no unambiguous way of extracting the off–shell behaviour of form functions from on–shell matrix elements. The ultimate reason is that the form functions of Eq. (2.4) are not only model dependent but also representation dependent, i.e., two representations of the same theory result in the same observables but different form functions.

Finally, let us discuss the above derivation within a somewhat different approach which does not make use of a calculation within a specific model or theory. Such a discussion also serves to demonstrate that our interpretation is independent of the fact that we made use of ChPT at $O(p^4)$. For that purpose we follow Gell–Mann and Goldberger in their derivation of the low–energy theorem for Compton scattering [18], and split the most general invariant amplitude of $\gamma(\epsilon,k)+\pi^+(p_i)\to\gamma'(\epsilon',k')+\pi^+(p_f)$ into two classes A and B. Class A consists of the most general pole terms and class B contains the rest,

$$\mathcal{M} = \epsilon'_\nu M^{\nu\mu}\epsilon_\mu = \mathcal{M}_A + \mathcal{M}_B. \tag{4.6}$$

The original motivation in Ref. [18] for such a separation was to isolate those terms of $\mathcal{M}$ which have a singular behaviour in the limit $k,k'\to 0$. We write class A in terms of the most general expressions for the irreducible, renormalized vertices and the renormalized propagator,

$$M_A^{\nu\mu} = -ie^2\Gamma^\nu(p_f,p_f+k')\Delta_R(p_i+k)\Gamma^\mu(p_i+k,p_i) + (k\leftrightarrow -k',\mu\leftrightarrow\nu), \tag{4.7}$$

where we made use of crossing symmetry. For sufficiently low energies class B can be expanded in terms of the relevant kinematical variables,

$$M_B^{\nu\mu} = a^{\nu\mu}(P) + b^{\nu\mu\rho}(P)k_\rho + c^{\nu\mu\rho}(P)k'_\rho + \ldots, \tag{4.8}$$

where $P = p_i + p_f$. Furthermore, in class A we expand the vertices and the renormalized propagator of the pion around their respective on–shell points, $p^2 = M^2$. We obtain for the propagator

$$\Delta_R^{-1}(p^2) = p^2 - M^2 - \Sigma(p^2) = (p^2 - M^2)(1 - \frac{p^2 - M^2}{2}\Sigma''(M^2) + \ldots), \tag{4.9}$$

where we made use of the standard normalization conditions $\Sigma(M^2) = \Sigma'(M^2) = 0$. The expansion of, e.g., the vertex describing the absorbtion of the initial photon in the s channel reads

$$\begin{aligned}\Gamma^\mu(p_i + k, p_i) &= (P^\mu + k'^\mu)F(0, M^2 + (s - M^2), M^2) \\ &= (P^\mu + k'^\mu)(1 + (s - M^2)\partial_2 F(0, M^2, M^2) + \ldots),\end{aligned} \tag{4.10}$$

where we made use of $k^2 = 0$, and where ∂_2 refers to partial differentiation with respect to the second argument. Inserting the result of Eqs. (4.9) and (4.10) into Eq. (4.7) we obtain for the s channel

$$\begin{aligned}M_s^{\nu\mu} &= -ie^2(P^\nu + k^\nu)(1 + (s - M^2)\partial_3 F(0, M^2, M^2) + \ldots) \\ &\times \frac{1}{s - M^2}(1 + \frac{s - M^2}{2}\Sigma''(M^2) + \ldots) \\ &\times (P^\mu + k'^\mu)(1 + (s - M^2)\partial_2 F(0, M^2, M^2) + \ldots) \\ &= -ie^2\frac{(P^\nu + k^\nu)(P^\mu + k'^\mu)}{s - M^2} + O((s - M^2)^0) \\ &= \text{“free” s channel + analytical terms},\end{aligned} \tag{4.11}$$

and an analogous term for the u channel. In Eq. (4.11) “free” s channel refers to a calculation with on–shell vertices. From Eq. (4.11) we immediately see that off–shell effects resulting from either the form functions or the renormalized propagator are of the same order as analytical contributions from class B. In the total amplitude off–shell contributions from class A cannot uniquely be separated from class B contributions. In the language of field transformations this means that contributions to $\mathcal{M}$ can be shifted between different diagrams leaving the total result invariant.

V. CONCLUSIONS

We discussed the electromagnetic vertex of an off–shell pion. The most general form of this vertex is constrained by symmetry principles such as Lorentz covariance, gauge invariance, time–reversal and charge–conjugation symmetry (see items 1) – 4) of Sec. II). In order to illustrate how such an off–shell vertex enters a covariant calculation of a scattering amplitude, we considered the specific example of Compton scattering from a pion. The calculation

was performed within the framework of chiral perturbation theory at $O(p^4)$. We generated an infinite number of equivalent representations of the chiral Lagrangian by making use of the concept of field transformations. This approach allowed us to compare the results of different representations of the same microscopic theory.

It was demonstrated that different but physically equivalent representations generate different off–shell electromagnetic vertices. On the other hand, all representations result in the same Compton scattering amplitude. This is a consequence of the equivalence theorem. As a result of our specific example we conclude that even in the framework of the *same* microscopic theory, given in different representations, it is not possible to uniquely extract the contributions to the scattering amplitude which result from off–shell effects in the pole terms.

In the language of Gell–Mann and Goldberger, by a change of representation, contributions can be shifted from class A to class B within the *same* theory. We can also express this differently; what appears to be an off–shell effect in one representation results, for example, from a contact interaction in another representation. In this sense, off–shell effects are not only model dependent, i.e., different models generate different off–shell form functions, but they are also representation dependent which means that even different representations of the same theory generate different off–shell form functions. This has to be contrasted with on–shell S–matrix elements which, in general, will be different for different models (model dependent), but always the same for different representations of the same model (representation independent).

We have seen that the most general result for the Compton scattering amplitude up to $O(p^4)$ can be obtained in a representation ($\beta_1 = 0$) with no off–shell effects at all in the electromagnetic vertex for real photons. This is a special feature of the momentum expansion of chiral perturbation theory up to $O(p^4)$, and one should not generalize this observation to higher orders in the momentum expansion. Higher–loop diagrams may, in general, yield off–shell contributions which cannot be transformed away.

In conclusion, the freedom of performing field transformations allows to shift contributions between different building blocks in different representations of the same theory, while the on–shell S–matrix remains the same. In general, quantum field theoretical models will yield off–shell vertices, however, they are not unique. In particular, they are not only model dependent but also representation dependent.

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